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Tessellated shading streaming - Supplementary Material

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1. Method

In this section we will present the full derivation of the equations mentioned in the main paper.

1.1. Spawning arbitrary amount of samples

We show how to control the 4 tessellation levels in order to spawn an arbitrary amount of samples within the bounds of the tessellator (up to 3169).

We will use the relationship between the tessellation levels and the amount of vertices spawned (equation 1). This has been already shown in the main paper and is mentioned here for convenience:

$$v(l) = 3\left(\left\lfloor \frac{l}{2} \right\rfloor + 1\right)\left(\left\lfloor \frac{l}{2} \right\rfloor + l \bmod 2\right) + (l+1) \bmod 2 \tag{1}$$

We first find the inner tessellation level required to spawn the number of samples n:

$$i_{lvl}(n) = l, \qquad v(l) <= n \qquad \wedge \qquad v(l+1) > n \tag{2}$$

Then, we add the contribution by the outer levels. Since each outer level can contribute with up to 64 samples, we just calculate the necessary number of samples and distribute them in increasing order

$$o_{lvl1}(n) = \min(n - i_{lvl}(n), 64) \tag{3}$$

$$o_{lvl2}(n) = \min(n - i_{lvl}(n) - o_{lvl1}(n), 64) \tag{4}$$

$$o_{lvl3}(n) = \min(n - i_{lvl}(n) - o_{lvl1}(n) - o_{lvl2}(n), 64) \tag{5}$$

Thus, the tessellation levels $\mathbf{L} = \begin{bmatrix} l_{o0} & l_{o1} & l_{o2} & l_i \end{bmatrix}$ for the required number of samples n are filled as follows:

$$\mathbf{L} = \begin{bmatrix} o_{lvl1}(n) & o_{lvl2}(n) & o_{lvl3}(n) & i_{lvl}(n) \end{bmatrix} \tag{6}$$

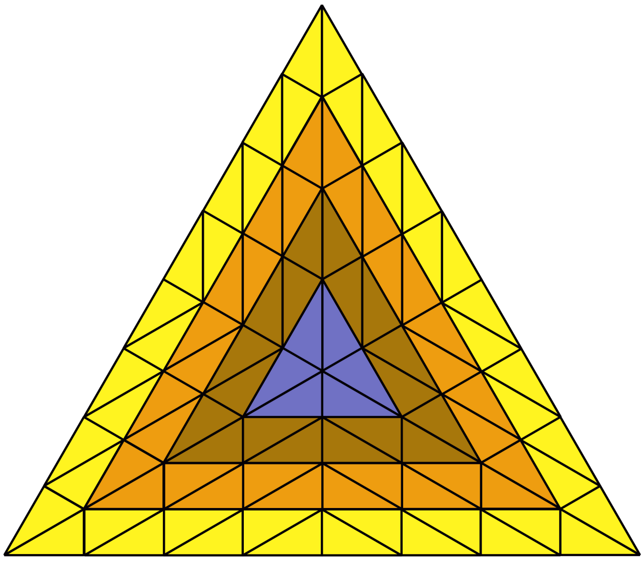


Figure 1: *The four layers of a tessellation pattern for all 4 tessellation levels set to 8.*

1.2. Determining the sample ID

Here we will show how to identify the unique sample Id for a vertex spawned by the tessellator. The known input parameters are the barycentric coordinates of the sample $\mathbf{B} = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix}$, sorted $\begin{bmatrix} \lambda_{min} & \lambda_{med} & \lambda_{max} \end{bmatrix}$ and tessellation levels $\mathbf{L} = \begin{bmatrix} l_{o0} & l_{o1} & l_{o2} & l_i \end{bmatrix}$, where l_{o0}, l_{o1}, l_{o2} stand for outer tessellation levels and l_i is inner tessellation level.

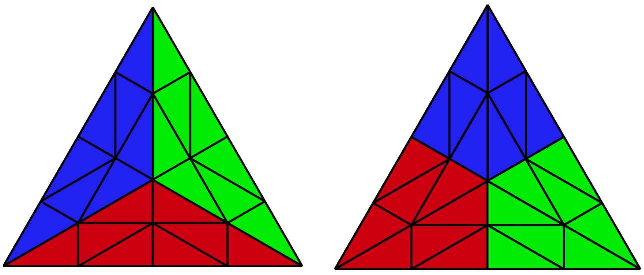
We utilize the inner distance introduced in the main paper $i(l)$ from equation 7:

$$o(l) = \frac{1}{l} \qquad\qquad i(l) = \frac{2}{3l} \tag{7}$$

and vertex count $v(l)$ from equation 1.

In order to compute the sample Id, we introduce a concept of layers within a tessellated triangle, shown in figure 1.

During the layer identification and within-layer sample identi-



(a) *Side identification* **(b)** *Tip identification*

Figure 2: *Side identification and Tip identification used in Oversampling and L-packing respectively.*

fication we utilize the fact of determining the to which tip and to which side of the triangle given sample belongs based on its barycentric coordinates (figure 2) We first split the triangle into layers according to the inner circles formed by tessellation. We identify the last layer as:

$$l'(\mathbf{L}) = \left\lfloor \frac{l_i - 1}{2} \right\rfloor \tag{8}$$

The exact layer of a sample can be identified as

$$l(\mathbf{B},\mathbf{L}) = \frac{\lambda_{min}}{i(l_i)} \tag{9}$$

The order of the sample is calculated as an order within the current layer plus an offset of a prefix sum of all the previous layers. The previous layer sample count can be calculated as:

$$p(\mathbf{B},\mathbf{L}) = v\Big(2\big(l'(\mathbf{L}) - l(\mathbf{B},\mathbf{L})\big) + \Big\lfloor \frac{l_o + 1}{2} \Big\rfloor\Big) \tag{10}$$

Next we calculate the order within the current layer. First we need to identify the side of the triangle to which the sample belongs (see figure 2), based on the values of the barycentric coordinates

$$h(l_0, l_1, d, n) = \begin{cases} 1, & |l_0 - dn| <= 0 \wedge |l_0 - l_1| >= 0 \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

$$S(\mathbf{B},\mathbf{L}) = \begin{cases} 0, & h(l_{o0}, l_{o2}, i(l_i), l(\mathbf{B},\mathbf{L})) = 1 \\ 1, & h(l_{o1}, l_{o0}, i(l_i), l(\mathbf{B},\mathbf{L})) = 1 \\ 2, & h(l_{o2}, l_{o1}, i(l_i), l(\mathbf{B},\mathbf{L})) = 1 \end{cases} \tag{12}$$

Then, we calculate the order within side:

$$os(\mathbf{B},\mathbf{L},t) = \big\lfloor \big(t - l(\mathbf{B},\mathbf{L}) i(l_i)\big) l_i \big\rceil \tag{13}$$

and determine offset the sample order based on the size of the current inner layer side

$$sl(\mathbf{B},\mathbf{L}) = l_i - 2l(\mathbf{B},\mathbf{L}) \tag{14}$$

Thus, we get the required sample Id for the inner layer:

$$id_i(\mathbf{B},\mathbf{L}) = \begin{cases} os(\mathbf{B},\mathbf{L},l_{o1}), & S(\mathbf{B},\mathbf{L}) = 0 \\ os(\mathbf{B},\mathbf{L},l_{o2}) + sl(\mathbf{B},\mathbf{L}), & S(\mathbf{B},\mathbf{L}) = 1 \\ os(\mathbf{B},\mathbf{L},l_{o0}) + sl(\mathbf{B},\mathbf{L}) \cdot 2, & S(\mathbf{B},\mathbf{L}) = 2 \end{cases} \tag{15}$$

Since the first layer is the outer-most layer within the triangle, its sample orders are dependent on the outer tessellation levels and we must treat this case separately:

$$id_o(\mathbf{B},\mathbf{L}) = \begin{cases} \lfloor \frac{l_{o1}}{o(l_{o0})} \rceil, & S(\mathbf{B},\mathbf{L}) = 0 \\ \lfloor \frac{l_{o2}}{o(l_{o1})} \rceil + l_{o1}, & S(\mathbf{B},\mathbf{L}) = 1 \\ \lfloor \frac{l_{o0}}{o(l_{o2})} \rceil + l_{o1} + l_{o2}, & S(\mathbf{B},\mathbf{L}) = 2 \end{cases} \tag{16}$$

Thus, the current layer order of a sample:

$$o(\mathbf{B},\mathbf{L}) = \begin{cases} id_o(\mathbf{B},\mathbf{L}), & layer(u,v,w,l_o) = 0 \\ id_i(\mathbf{B},\mathbf{L}), & \text{otherwise} \end{cases} \tag{17}$$

And the final sample order within the whole patch we get as a sum of the sample location within the current layer and all the previous layers

$$O(\mathbf{B},\mathbf{L}) = o(\mathbf{B},\mathbf{L}) + p(\mathbf{B},\mathbf{L}) \tag{18}$$

1.3. Sample ID to new barycentric coordinates

In this section we derive the formulas used to map from sample ID to new barycentric coordinates of a triangle used in Oversampling method. We will use the block height R and block count P functions introduced in the main paper:

$$R(w,h) = \left\lceil \frac{w}{h} \right\rceil \qquad P(w,h) = \frac{h}{R(w,h)} \tag{19}$$

For convenience we will denote $R(w,h)$ as α and $P(w,h)$ as β.

We introduce a helper function:

$$b(I,w,h) = \frac{-\sqrt{\alpha(-8I + \alpha(2w+3)^2 + 8(w+1)} + 2\alpha w + 3\alpha}{2\alpha} \tag{20}$$

Then, we need to get the number of samples in all the previous blocks as:

$$b_P(I,w,h) = \beta(w + 2 - \frac{\beta+1}{2})b(I,w,h) + w + 1 \tag{21}$$

Note that this is a non-recursive way to compute a prefix sum over the sample count up to a given layer, which is very friendly to parallel implementations on GPU. Computing recursively or within a loop would degrade performance due to thread divergence.

Next, we identify the block width as:

$$b_w(I,w,h) = w - b(I,w,h) + 1 \quad (22)$$

and get the index of the sample within the current block.

$$b_{id}(I,w,h) = I - b_p(I,w,h) \quad (23)$$

The next steps for mapping from x, y coordinates to new barycentric coordinates of the sample is mentioned in the main paper.